AXIOM YOUNG RESEARCH

Original Research
Articles in

International Journal of Student Mathematics & Science

Mathematics
Applied Science
Computational Thinking-
Scientific Modeling

Volume 3

A Platform for the
Next Generation
of Scientists

Axiom
Journal

INTERNATIONAL JOURNAL

AXIOM YOUNG RESEARCH

International Journal of Student Mathematics and Science

VOLUME 3

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A PLATFORM FOR THE NEXT GENERATION OF SCIENTISTS

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MATHEMATICS · APPLIED SCIENCE

SCIENTIFIC MODELING

A PLATFORM FOR THE NEXT GENERATION OF SCIENTISTS

Where curiosity becomes discovery.

EXPLORING IDEAS. ADVANCING YOUNG SCIENCE.

> *Axiom Young Research is an international journal dedicated to publishing original research conducted by young scholars in mathematics, science, and interdisciplinary STEM fields. The journal provides a professional platform where students can present rigorous ideas, develop scientific thinking, and share their discoveries with the global academic community. By encouraging curiosity, logical reasoning, and independent inquiry, Axiom Young Research seeks to inspire the next generation of scientists and mathematicians.*

Editorial Note

VOLUME 3 EDITION

The advancement of science and mathematics depends on curiosity, creativity, and rigorous thinking. These qualities are not limited by age. Many important discoveries in the history of science began with simple questions asked by young minds.

Axiom Young Research was established to provide a professional platform where young scholars can present original ideas and develop their research abilities through the process of academic writing and publication.

This journal encourages students to move beyond routine problem solving and to engage in deeper exploration of mathematical structures, scientific phenomena, and computational ideas. Through independent research, students learn how to formulate questions, construct logical arguments, analyze data, and communicate results clearly.

The articles presented in this volume reflect the intellectual curiosity and dedication of students from diverse backgrounds who share a passion for discovery. Each contribution represents not only a research result but also a step in the development of future scientists and scholars.

We hope that this journal will inspire more students to pursue independent inquiry and contribute to the global scientific community.

Editorial Committee
Axiom Young Research

JOURNAL FOUNDATION

Introduction

Scientific discovery often begins with a simple question.

> *Why does this pattern occur?*
>
> *Is there a hidden structure behind this phenomenon?*
>
> *Can mathematics explain what we observe in the world?*

Many of the most important ideas in science and mathematics started with curiosity and careful observation. Axiom Young Research was created to support young scholars who are eager to explore such questions and to transform their ideas into structured research.

The journal encourages students to move beyond traditional classroom learning and engage in independent investigation. By developing hypotheses, constructing mathematical arguments, building models, and analyzing data, students gain experience in the authentic practice of scientific inquiry.

Axiom Young Research provides a platform where these ideas can be shared with a broader academic community.

CORE PRINCIPLES

Our Philosophy

The philosophy of Axiom Young Research is founded on several core principles.

CURIOSITY DRIVES DISCOVERY

Scientific and mathematical exploration begins with curiosity. When students ask deeper questions and seek logical explanations, they begin the journey toward genuine research.

IDEAS HAVE NO AGE LIMIT

Important insights can emerge at any stage of learning. Young researchers often bring fresh perspectives and creative approaches to problems that have been studied for generations.

RIGOR MATTERS

Even at the student level, research should be guided by clear reasoning, careful analysis, and intellectual honesty. Axiom Young Research encourages students to develop these habits early in their academic journey.

ACCESS TO RESEARCH SHOULD BE OPEN

Opportunities for scholarly publication should not be limited by financial barriers or institutional privilege. The journal strives to make academic participation accessible to students from diverse backgrounds around the world.

THE FUTURE OF SCIENCE BEGINS WITH STUDENTS

Today’s young researchers will become tomorrow’s scientists, mathematicians, engineers, and innovators. By providing a platform for early academic exploration, Axiom Young Research supports the development of the next generation of thinkers and problem solvers.

MESSAGE FROM THE BOARD

Head Director's Message

The beginning of every meaningful discovery lies in a question.

Young students often possess an extraordinary curiosity about the world. They observe patterns, challenge assumptions, and attempt to explain complex ideas using the tools of mathematics and science. Unfortunately, many of these ideas remain confined to classrooms or personal notebooks, never reaching a wider audience.

Axiom Young Research was established to change that.

Our goal is to provide talented students with the opportunity to experience the process of academic research and publication. Writing a research paper requires more than simply presenting results; it demands careful reasoning, clear communication, and intellectual discipline. Through this process, students learn how to develop ideas, evaluate evidence, and present arguments in a scholarly manner.

We believe that intellectual curiosity should be encouraged at every stage of education. By supporting young researchers and providing them with a professional platform to share their work, Axiom Young Research hopes to inspire a lifelong engagement with mathematics, science, and discovery.

We are honored to present the work of young scholars who represent the future of scientific and mathematical inquiry.

HEAD DIRECTOR

Hemant Kumar Singh

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Axiom Journal

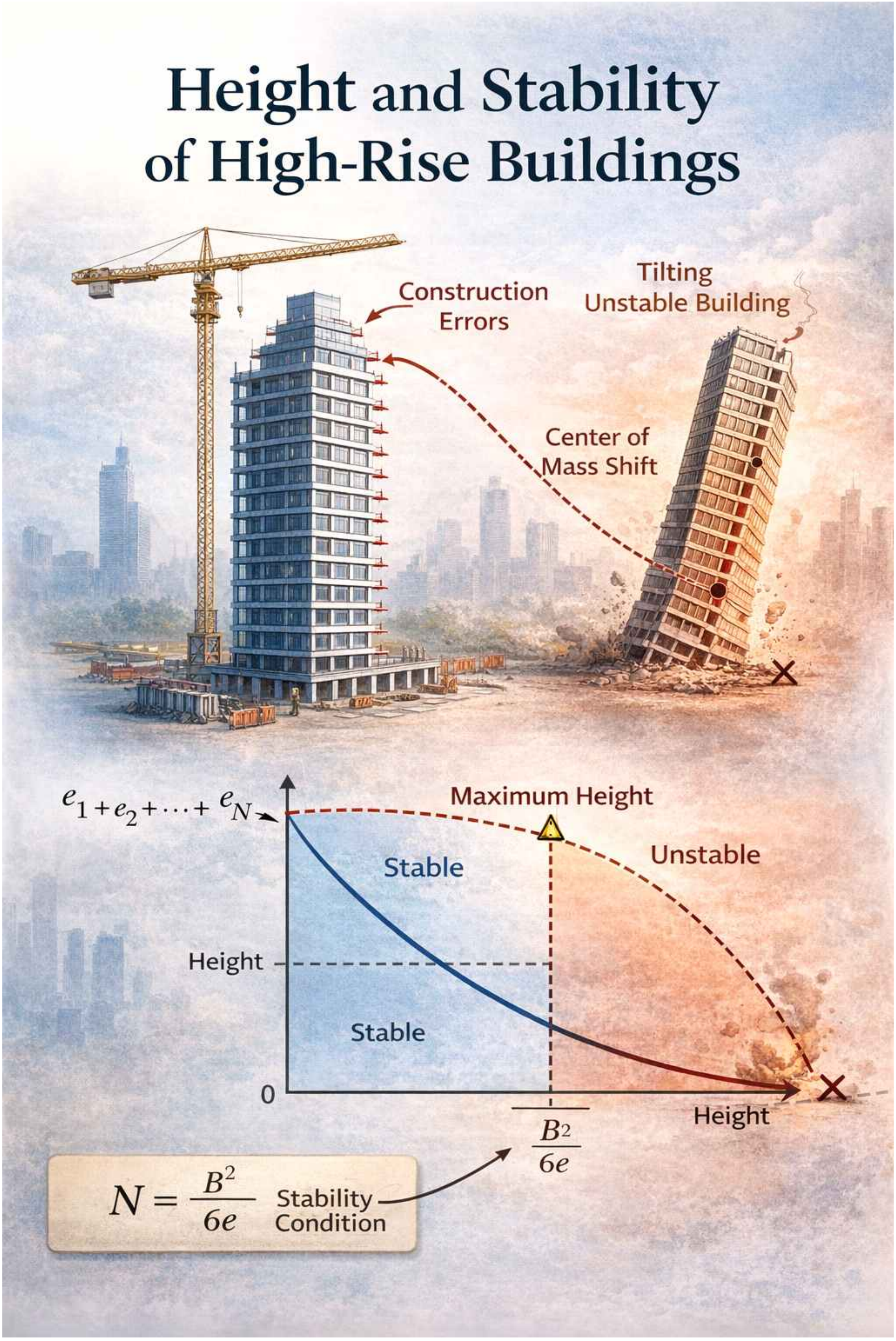
A Mathematical Study on the Relationship Between the Height and Stability of High-Rise Buildings

A Mathematical Study on the Relationship Between the Height and Stability of High-Rise Buildings

Abstract

This study investigates the theoretical relationship between the height of a building and its structural stability using a simplified mathematical model. Each floor of the building is represented as a rectangular block with small construction errors on its left and right walls. As the number of floors increases, these tiny errors accumulate and cause a gradual tilt in the structure. By calculating the displacement of the building's center of mass and setting a stability condition based on the base width, we derive the maximum number of floors that a building can have before becoming unstable. The result shows that the allowable height increases with a wider base, lower floor height, and smaller construction error. This work provides a clear mathematical explanation for how precision in construction directly limits the achievable height of high-rise buildings.

Keywords: Building Stability, Construction Error, Center of Mass, Maximum Height, Structural Modeling


Height and Stability
of High-Rise Buildings
Construction
Errors
Tilting
Unstable Building
Center of
Mass Shift
$e_1 + e_2 + \cdots + e_N$
Maximum Height
Stable
Unstable
Height
Stable
0
Height
$\frac{B^2}{6e}$
$N = \frac{B^2}{6e}$
Stability
Condition

1. Introduction

Throughout history, humanity has continuously developed its civilization while steadily increasing its population. The development of agriculture allowed a stable supply of food, and advances in medicine greatly reduced deaths from disease. After the Industrial Revolution, rapid urbanization raised both productivity and living standards, which together accelerated population growth.

As a result, the number of people on Earth became far greater than the amount of land that humans can actually live on. In many cities, limited land had to support an ever-growing population, which made it necessary to expand living space not horizontally but vertically. In other words, using limited space efficiently became one of the most important challenges for humankind.

In particular, after the 20th century, the development of reinforced concrete structures and design technologies capable of withstanding wind and earthquake loads led to the construction of skyscrapers in many parts of the world (refer to [3, 4]). These buildings became more than just living spaces; they turned into symbols of cities and are regarded as masterpieces that represent the achievements of modern architectural engineering.

The tallest building in the world today is the Burj Khalifa, located in Dubai, United Arab Emirates. It reaches a height of about 828 meters and has 163 floors, which makes it the tallest man-made structure ever built.

Figure 1. The Burj Khalifa, designed by Adrian Smith, is the tallest building in the world, reaching 828 meters with 163 floors. (Source: Architectural Digest [1])

No matter how much construction technology advances, it is impossible to make the error in the building process completely zero. During the process of stacking each floor, very small horizontal or vertical errors occur due to several factors such as slight deformation of materials, vibrations of construction equipment, and alignment errors made by workers. These errors are almost invisible in a single floor, but as the number of floors increases to dozens or even hundreds, they accumulate and cause the central axis of the entire structure to gradually deviate from its foundation.

Eventually, these accumulated errors shift the center of mass of the building, which directly affects the overall stability of the structure. No matter how strong the materials are, if the center of mass moves too far away from the base, the building may tilt or, in extreme cases, collapse.

In this study, we represent the effect of cumulative construction errors using a simple mathematical model and theoretically calculate the maximum height at which a building can remain stable without collapsing, assuming that a constant error occurs continuously.

This study is organized as follows. In Section 2, we mathematically model a high-rise building and explain in detail what kinds of errors can occur during the actual construction process. In Section 3, we examine how these errors affect the structure as more floors are stacked by calculating the center vectors of each floor’s midpoint. Section 4 deals with the conditions under which a building can remain stable from a physical point of view, presenting criteria that prevent the building from tilting or collapsing based on the relationship between the center of mass and the foundation. In Section 5, using the center vectors of the floors introduced in Section 3 and the stability condition from Section 4, we calculate the maximum number of floors the building can have. Finally, Section 6 summarizes the overall results of this study.

2. Analysis of Each Floor of the Building

In this section, we will explore how to model a building in physical terms. A real building exists in three dimensions, so its length, width, and height must all be considered. However, since the errors caused by length and those caused by width can be assumed to be independent of each other, we will simplify the problem by assuming that the building has a rectangular shape lying in a two-dimensional plane.

Now, each floor of the building will be represented by a rectangular block. Let the horizontal length be denoted by L and the vertical height by H. Then, we can think of the building as being constructed by stacking many such rectangular blocks on top of one another.

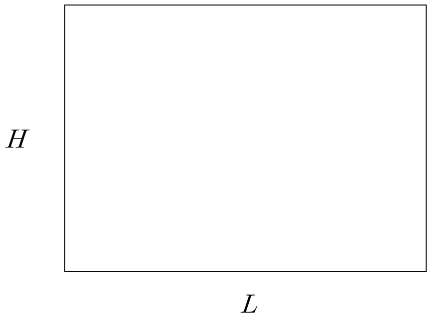


Figure 2 A rectangular block with width L and height H.

Of course, in a perfect situation with no errors, stacking an infinite number of such blocks would still maintain stability. However, reality is not like that. No matter how precisely the walls are built, it is impossible to make the heights of the left and right walls exactly equal to H. Therefore, let the actual heights of the left and right walls be $H+a$ and $H+b$, respectively. Here, a and b represent the small errors in the wall heights. If a or b is negative, it means that the wall was built slightly shorter than the target height H; if it is positive, the wall is slightly taller than the target height.

When the heights of the left and right walls are not equal, the block is no longer a perfect rectangle. Instead, it becomes a trapezoid whose left and right sides are not parallel in height. This trapezoid can be illustrated as shown in the following figure.

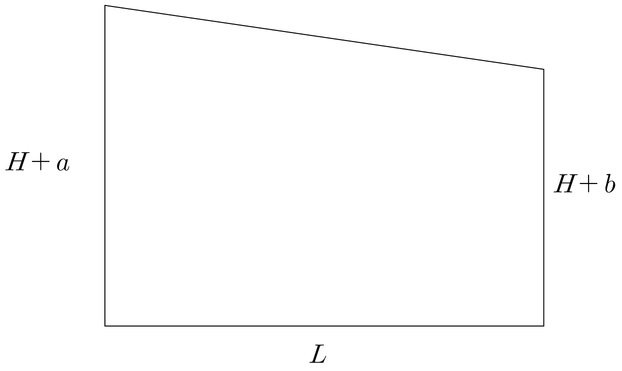


Figure 3 A block with horizontal length L, left wall height $H+a$, and right wall height $H+b$. In this figure, the left wall is drawn slightly longer, so the top edge is rotated a little clockwise relative to the bottom edge. In this case, θ can be considered a small positive value.

The angle formed between the lower base and the upper base of this trapezoid can be easily calculated by drawing an auxiliary line. Since

$$\tan\theta = \frac{(H+a)-(H+b)}{L} = \frac{a-b}{L},$$

we have

$$\theta = \arctan\left(\frac{a-b}{L}\right).$$

If θ is positive, it means that the bottom surface is slightly rotated in the clockwise direction, while if θ is negative, the bottom surface is slightly rotated in the counterclockwise direction.

Let us now assume that the first floor has been completed as shown in Figure 2. Next, we need to stack the second, third, and higher floors on top of it, using blocks of the same shape. However, suppose that the columns are built vertically from the ground rather than following the slightly tilted ceiling of the first floor. In this case, even though the ceiling of the first floor is not perfectly horizontal, the second floor is placed on it as if it were the base. As a result, the entire structure tilts by an angle of θ, and the columns of the next floor are rotated by the same angle, as illustrated in the following figure.

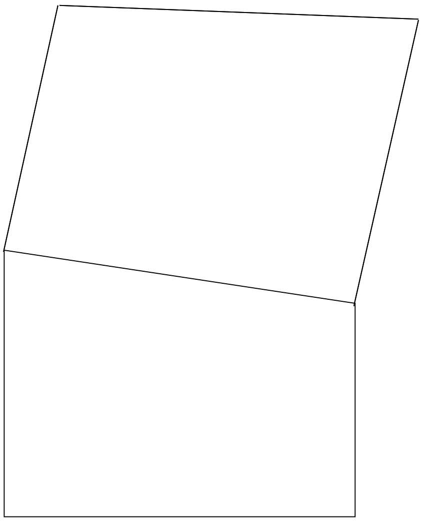


Figure 4 A new floor stacked on top of a base that is not perfectly horizontal.

In the next section, we will show inductively how the building is stacked when N floors are added, following the same method of placing one floor on top of another as defined earlier.

3. Accumulation of Errors Through Multiple Floors

To analyze the building that is stacked inductively, we assume the following variables.

$a_1,\ a_2,\ \ldots\ ,\ a_N$: the error of the left wall on each floor,

$b_1,\ b_2,\ \ldots\ ,\ b_N$: the error of the right wall on each floor.

We will now use these errors to express several geometric quantities of the building, such as lengths and positions. First, let us define the physical quantities that we want to study:

$L_1,\ L_2,\ \ldots\ ,\ L_N$: the length of the bottom edge of each floor,

$\theta_1,\ \theta_2,\ \ldots\ ,\ \theta_N$: the angle between the ceiling and the floor of each floor.

The signs of $\theta_1,\ \theta_2,\ \ldots\ ,\ \theta_N$ follow the sign convention for θ defined in the previous section. Finally, let us denote the coordinates of the center point of each floor's base by

$$\vec{x}_1,\ \vec{x}_2,\ \ldots\ ,\ \vec{x}_N.$$

Here $\vec{x}_k$ is a two-dimensional vector that gives the position of the center of the k-th floor's base.

We can now make initial assumptions for L_1 and $\vec{x}_1$ in order to compute the above variables inductively. Since we can place the center of the first floor's base at the origin, we set $\vec{x}_1 = (0,0)$. Also, because the first floor is built on a flat ground, its horizontal width can be set exactly to L. Thus it is reasonable to take $L_1 = L$.

Following the same process used to compute θ in the previous section, we can calculate θ_1 as

$$\theta_1 = \arctan\left(\frac{a_1 - b_1}{L_1}\right).$$

In fact, this formula also holds for all θ_2, ... , θ_N. For every $1 \leq k \leq N$:

$$\theta_k = \arctan\left(\frac{a_k - b_k}{L_k}\right)$$

from a simple geometric argument.

We have not yet derived an inductive formula. The expression for θ_k above depends only on a_k, b_k, and L_k, so it holds only for each individual floor. To use an inductive relation, let us find the relationship between L_k and L_{k+1} for all $1 \leq k \leq N-1$

First, L_k is the length of the floor at the k-th level, and L_{k+1} is the length of the floor at the $k+1$-th level. At the same time, L_{k+1} is also the length of the ceiling of the k-th level.

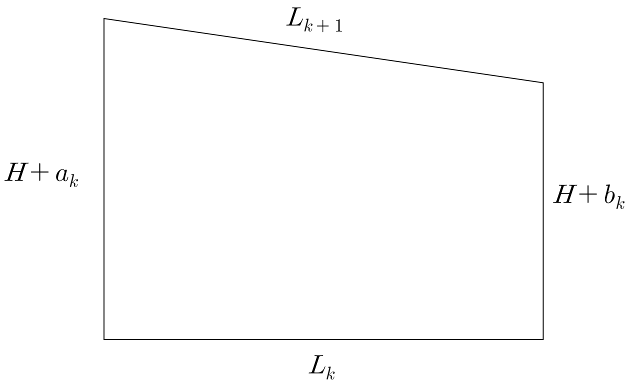


Figure 5 A schematic diagram of the k-th floor. The length of the floor is L_k, the length of the ceiling is L_{k+1}, and the lengths of the left and right walls are $H+a_k$ and $H+b_k$, respectively.

From one end of L_{k+1}, the perpendicular dropped to the opposite wall has length L_k, and the vertical difference is $|a_k - b_k|$. By the Pythagorean theorem we obtain

$$L_{k+1}^2 = L_k^2 + (a_k - b_k)^2.$$

If we use this relationship, we get

$$L_k^2 = L_{k-1}^2 + (a_{k-1} - b_{k-1})^2 = \cdots = L_1^2 + \sum_{j=1}^{k-1} (a_j - b_j)^2.$$

From $L_1 = L$, we can finally conclude that

$$L_k = \left(L^2 + \sum_{j=1}^{k-1}(a_j - b_j)^2\right)^{\frac{1}{2}} \text{ for all } 1 \le k \le N.$$

Furthermore, we can express θ_k from the above result as follows:

$$\theta_k = \arctan\left(\frac{a_k - b_k}{\left(L^2 + \sum_{j=1}^{k-1}(a_j - b_j)^2\right)^{\frac{1}{2}}}\right) \text{ for all } 1 \le k \le N.$$

Therefore, for all $1 \le k \le N$, we have obtained explicit expressions for L_k and θ_k. Using these expressions, we will now compute the center vectors of the base of each floor.

Let $\overrightarrow{v_k}$ be the vector that connects the center of the floor to the center of the ceiling on the k-th level $1 \le k \le N$. The length of $\overrightarrow{v_k}$ equals the average of the left and right wall heights, which is

$$H + \frac{a_k + b_k}{2}.$$

This is because the left wall, $\overrightarrow{v_k}$, and the right wall are all oriented in the direction perpendicular to the floor. Therefore, among the length and direction needed to determine the vector, we have already found the length.

Now let us consider the direction of this vector. For the first floor, the direction of $\overrightarrow{v_1}$ is perpendicular to the ground. For the second floor, however, the direction of $\overrightarrow{v_2}$ is perpendicular to the ceiling of the first floor. Since the ceiling of the first floor is rotated by θ_1 relative to the first floor's base, $\overrightarrow{v_2}$ is also rotated by θ_1 relative to $\overrightarrow{v_1}$. As defined earlier, if θ_1 is positive, the rotation is clockwise, and if it is negative, the rotation is counterclockwise.

However, this rotation angle accumulates as the floors go up. For example, the ceiling of the second floor is rotated by θ_2 with respect to the second floor's base. In other words, the second floor's ceiling is rotated by $\theta_1+\theta_2$ with respect to the first floor's base. By using the same logic inductively, for any natural number $1 \leq k \leq N-1$, the ceiling of the k-th floor is rotated by $\theta_1+\theta_2+\cdots+\theta_k$ with respect to the first floor's base. Therefore, the base of the $k+1$-th floor is also rotated by $\theta_1+\theta_2+\cdots+\theta_k$ with respect to the first floor's base.

Hence the direction of $\vec{v}_{k+1}$ is the direction obtained by rotating $\vec{v}_1$, which points vertically upward, by $\theta_1+\theta_2+\cdots+\theta_k$. The unit vector of this direction is

$$(\sin(\theta_1+\cdots+\theta_k), \sin(\theta_1+\cdots+\theta_k)).$$

We multiply the length of v_{k+1} to the above unit vector to get the explicit form of $\vec{v}_{k+1}$ as follows:

$$\vec{v}_{k+1} = \left(H+\frac{a_{k+1}+b_{k+1}}{2}\right)(\sin(\theta_1+\cdots+\theta_k), \sin(\theta_1+\cdots+\theta_k)) \text{ for } 1 \leq k \leq N-1.$$

Especially, we have

$$\vec{v}_1 = \left(H+\frac{a_1+b_1}{2}\right)(0,1).$$

To simplify the calculation, we assume that

$$a_k+b_k=0 \text{ for all } 1 \leq k \leq N.$$

This assumption is quite valid since the instability of the building caused by the difference between two error a_k-b_k. If a_k and b_k are the same, then we do not need to consider them as errors. If $a_k=b_k$, then we can consider it as we built a higher floor compared to the original plan. In other words, we assume that the average of the error is zero since we only want to consider factors for the instability.

Then, we can simplify the form of vectors as follows:

$$\overrightarrow{v}_1 = H(0,1) \text{ and } \overrightarrow{v}_{k+1} = H(\sin(\theta_1 + \cdots + \theta_k), \sin(\theta_1 + \cdots + \theta_k)) \text{ for } 1 \leq k \leq N-1.$$

From the definition of $\overrightarrow{x}_1$, $\overrightarrow{x}_2$, ... , $\overrightarrow{x}_N$ and $\overrightarrow{v}_1$, $\overrightarrow{v}_2$... , $\overrightarrow{v}_{N-1}$, for any $1 \leq k \leq N-1$ we get

$$\overrightarrow{x}_{k+1} - \overrightarrow{x}_k = \overrightarrow{v}_k.$$

Therefore, we can find $\overrightarrow{x}_2$, ... , $\overrightarrow{x}_N$ as follows:

$$\overrightarrow{x}_{k+1} = \overrightarrow{v}_1 + \cdots + \overrightarrow{v}_k \text{ for all } 1 \leq k \leq N-1.$$

Especially, we have $\overrightarrow{x}_1 = (0,0)$.

In the next section, we will discuss the conditions under which the building becomes stable. By substituting the vectors $\overrightarrow{x}_1$, $\overrightarrow{x}_2$, ... , $\overrightarrow{x}_N$ into the model, we will examine under what conditions the entire structure satisfies the stability requirement.

4. Physical Interpretation of the Stability Condition

In this section, we will give a physical explanation of what condition the midpoint of each floor’s base must satisfy for the building to remain stable, and we will substitute the actual values of $\vec{x}_1$, $\vec{x}_2$, ... , $\vec{x}_N$ to verify this condition.

First, when stacking the floors, the bottom part of each block carries the greatest weight. Therefore, we can assume that the center of mass of each floor’s block is concentrated near its base. If the center of mass of any block goes beyond the base area of the building, that block becomes unstable.

The base of the building corresponds to the range of the first floor’s bottom surface. Therefore, the horizontal range of the building’s base is

$$-\frac{L}{2} \leq x \leq \frac{L}{2}.$$

If the horizontal coordinate of $\vec{x}_k$ for any floor lies outside this range, it means that the building has become unstable. In other words, the condition for the k-th floor to become unstable is that the absolute value of the x-coordinate of $\vec{x}_k$ is greater than $\frac{L}{2}$.

For example, when $k=1$, the x-coordinate of $\vec{x}_1$ is 0. Therefore, its absolute value is always (0), which means the first floor is always stable.

For $1 \leq k \leq N-1$, to compute the x-coordinate of $\vec{x}_{k+1}$, we must add the x-coordinates of all the vectors $\vec{v}_1$, $\vec{v}_2$, ... , $\vec{v}_k$, because the following relation holds:

$$\vec{x}_{k+1} = \vec{v}_1 + \vec{v}_2 + \cdots + \vec{v}_k.$$

Recall that

$$\vec{v}_1 = H(0,1) \text{ and } \vec{v}_{k+1} = H(\sin(\theta_1 + \cdots + \theta_k), \sin(\theta_1 + \cdots + \theta_k)) \text{ for } 1 \leq k \leq N-1.$$

We can simply denote the above expressions as

$$\vec{v}_k = H\left(\sin\left(\sum_{j=1}^{k-1}\theta_j\right), \cos\left(\sum_{j=1}^{k-1}\theta_j\right)\right) \text{ for all } 1 \le k \le N$$

and its x-coordinate is $H\sin\left(\sum_{j=1}^{k-1}\theta_j\right)$. Finally, for any $1 \le k \le N$, the x-coordinate of x_k can be expressed as

$$H\sum_{l=1}^{k-1}\sin\left(\sum_{j=1}^{l-1}\theta_j\right).$$

Therefore, the condition for the block corresponding to the k-th floor to remain stable is as follows:

$$\left|H\sum_{l=1}^{k-1}\sin\left(\sum_{j=1}^{l-1}\theta_j\right)\right| < \frac{L}{2}.$$

This condition looks quite complicated because the sum appears inside a sine function. Therefore, let us use the following approximation that is often used in physics (refer [2]):

$$\sin x \simeq x \text{ for small } |x|.$$

Using this approximation, the stability condition derived earlier becomes much simpler.

$$\left|\sum_{l=1}^{k-1}\sum_{j=1}^{l-1}\theta_j\right| < \frac{L}{2H}.$$

We can simplify the double sum in the absolute value as follows:

$$\begin{aligned}\sum_{l=1}^{k-1}\sum_{j=1}^{l-1}\theta_j &= \sum_{l=1}^{k-1}(\theta_1+\theta_2+\cdots+\theta_{l-1}) \\ &= (k-2)\theta_1+(k-3)\theta_2+\cdots+\theta_{k-2}\end{aligned}$$

For $k=1$ or $k=2$, the above sum is zero since there are no indices l and j satisfying $1 \leq l \leq k-1$ and $1 \leq j \leq l-1$. From this double sum expression, we get

$$\left|(k-2)\theta_1 + (k-3)\theta_2 + \cdots + \theta_{k-2}\right| < \frac{L}{2H}.$$

Since $k=1$ and $k=2$ always satisfy the above condition, we can conclude that building the first floor and the second floor is always stable. However, we need to be careful when we build more than or equal to the third floor.

The explicit form of θ_1, ... , θ_N are given as follows:

$$\theta_k = \arctan\left(\frac{a_k - b_k}{\left(L^2 + \sum_{j=1}^{k-1}(a_j - b_j)^2\right)^{\frac{1}{2}}}\right) \text{ for all } 1 \leq k \leq N.$$

Now, we again use another approximation that is often used in physics (refer [2]):

$$\tan x \simeq x \text{ for small } |x|$$

which is equivalent to

$$x \simeq \arctan x \text{ for small } |x|.$$

Then, we have

$$\theta_k \simeq \frac{a_k - b_k}{\left(L^2 + \sum_{j=1}^{k-1}(a_j - b_j)^2\right)^{\frac{1}{2}}} \text{ for all } 1 \leq k \leq N.$$

Furthermore, compare to errors $a_j - b_j$ (for $1 \leq j \leq N$), L is extremely large. For instance, if the height of one floor is 2 m, then the error will be less than 1 cm or 0.5 cm. Therefore, we can further assume that

$$|L| \gg |a_j - b_j| \quad \text{for all } 1 \le j \le N$$

and we eventually get

$$\theta_k \simeq \frac{a_k - b_k}{L} \quad \text{for all } 1 \le k \le N.$$

To simplify the notation, we set

$$e_k = a_k - b_k \quad \text{for all } 1 \le k \le N.$$

If we substitute the above results into the stability condition

$$\left|(k-2)\theta_1 + (k-3)\theta_2 + \cdots + \theta_{k-2}\right| < \frac{L}{2H},$$

we get

$$\left|(k-2)e_1 + (k-3)e_1 + \cdots + e_{k-2}\right| < \frac{L^2}{2H}.$$

In the next section, we calculate the maximum number of the floors of the building using the above stability condition.

5. Maximum Number of Floors

In the previous section, we concluded that when the height differences at each floor are denoted by e_1, e_2, ... , e_N, the length of the building's base by (L), and the height of each floor by H, the condition for the k-th floor to remain stable is given as follows:

$$|(k-2)e_1+(k-3)e_2+\cdots+e_{k-2}|<\frac{L^2}{2H}.$$

If e_1, e_2, ... , e_N alternate between positive and negative values, the errors may cancel each other out and help the structure remain stable. However, when constructing a building, we must consider the worst possible case. What if, by coincidence, all e_1, e_2, ... , e_N have the same sign? In that case, one side of the wall would be built slightly higher each time, causing the entire building to gradually bend in the opposite direction.

If all the errors have the same sign, we can regard this as the worst-case scenario. Then, we let $e_1=e_2=\cdots=e_N=e$ and substitute this into the stability condition derived above.

$$|(k-2)e+(k-3)e+\cdots+e|=\frac{(k-1)(k-2)}{2}\,|e|\,<\frac{L^2}{2H}.$$

Therefore, the condition for the k-th floor to remain stable can be written as follows.

$$|e|<\frac{L^2}{(k-1)(k-2)H}.$$

If $k=1$ or $k=2$ then the above condition is always satisfied, and we can again check that the stability condition always holds for the first and second floors. For $k>2$, then the above condition should be satisfied and the maximum number of floor N is the maximum number N satisfying

$$|e|<\frac{L^2}{(N-1)(N-2)H}$$

and is equivalent to

$$(N-1)(N-2) < \frac{L^2}{|e|H}.$$

Furthermore, we can solve the above inequality as follows:

$$N < \frac{3}{2} + \sqrt{\frac{1}{4} + \frac{L^2}{|e|H}}.$$

Since N is a natural number, we only expressed the meaningful part of the inequality. Therefore, the maximum value of N can be expressed as follows:

$$N_{\max} = \left\lfloor \frac{3}{2} + \sqrt{\frac{1}{4} + \frac{L^2}{|e|H}} \right\rfloor,$$

where $\lfloor x \rfloor$ is the greatest integer not exceeding x. As we can see in the above expression, $N_{\max}$ is increased for larger L, smaller H, and smaller $|e|$ (maximum error). This result agrees with our intuition. The building becomes more stable when the base is wider, each floor is lower, and the construction error is smaller, allowing more floors to be safely stacked.

Now, in the next Conclusion section, we will discuss this result and summarize the overall findings of the study.

6. Conclusion

In this study, we developed a mathematical model to analyze how small construction errors affect the overall stability of a high-rise building. By representing each floor as a rectangular block with slight height differences between its left and right walls, we derived the angular deviation of each floor and showed how these errors accumulate as the building becomes taller. Using this accumulated tilt, we established a stability condition based on the position of the building’s center of mass relative to its base. From this, we derived an explicit formula for the maximum number of floors that can be safely stacked.

The results show that the stability of a building depends on three main factors: the base width, the height of each floor, and the magnitude of the construction error. Specifically, a wider base, lower floor height, and smaller construction error allow the structure to remain stable at greater heights. Although the model is simplified, it successfully explains how tiny errors in construction can fundamentally limit the achievable height of skyscrapers. This study demonstrates the importance of mathematical reasoning in understanding and predicting structural stability in real-world engineering.

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